Lexicographic-based Partially Preordered Removed Sets Revision

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Abstract

The revision of totally preordered belief bases has been extensively studied. However, in case of partial ignorance, pieces of information are partially preordered and few effective approaches of revision have been proposed. The paper presents a lexicographic-based revision operation for partially preordered belief bases defined within the framework of Partially Preordered Removed Sets Revision. This operation stems from the lexicographic preference between the removed sets (sets of beliefs to remove to restore consistency). It shows that this operation captures revision with memory and allows for implementing it thanks to ASP. An application to real data in the context of the VENUS european project is presented.

Introduction

In knowledge representation and reasoning for artificial intelligence, belief revision adresses the problem of incorporating new pieces of information in an agent's initial beliefs, in order to maintain consistency, while keeping new information and removing the least possible previous beliefs. A characterization of belief revision has been provided by Alchourron, Gärdenfors, Makinson (AGM) with a set of postulates that any revision operation should satisfy (Gärdenfors 1988). Katsuno and Mendelzon (KM) reformulated AGM's postulates and provided a representation theorem that characterizes revision operations based on total preorders (Katsuno and Mendelzon 1991). The revision of totally preordered information has been extensively studied and several revision operations have been proposed like possibilistic revision (Dubois and Prade 1992), (Dubois and Prade 1997) or adjustment revision (Williams 1995), linear-based revision (Nebel 1994), natural revision (Boutilier 1993), lexicographic-based revision (Benferhat et al. 1993; Nayak 1994), ranked revision (Lehmann 1995), revision with memory (Benferhat et al. 2000; Papini 2001; Konieczny and Perez 2000). Postulates have been proposed for iterated revision (Darwiche and Pearl 1997) and more recently several works focus on iterated revision (Booth and Meyer 2006; Jin and Thielscher 2007; Booth and Nittka 2008; Delgrande and Jin 2008). As pointed by (Delgrande, Dubois, and Lang 2006), the different approaches proposed in the literature for revising an epistemic state can be classified according to three different points of view. Given a plausibility ordering on interpretations describing the background knowledge and an new piece of information, belief revision as defeasible inference (BRDI) amounts to find the most plausible interpretation satisfying the input information, belief revision as incorporation of evidence (BRIE), amounts to change the plausibility ordering in presence of a new piece of information and belief revision of background knowledge (BRBK) means revising the background knowledge by a generic information.

Some approaches have been implemented (Williams and Williams 1997; Delgrande, Hunter, and Schaub 2002), among them, Removed Sets Revision which has been initially proposed in (Würbel, Jeansoulin, and Papini 2000) for revising a set of propositional formulae. This approach aims at inconsistency minimizing (Papini 1992; Hansson 1994) and stems from removing a minimal number of formulae, called removed set, to restore consistency. The Removed Sets Revision (RSR) and then a prioritized form of Removed Sets Revision, called Prioritized Removed Sets Revision (PRSR) (Ben-Naim et al. 2004) have been encoded into answer set programming and allowed for solving a practical revision problem coming from a real application in the framework of geographical information system.

However in some applications, an agent has not always a total preorder between pieces of information at his disposal, but is only able to define a partial preorder between them, particularly in case of partial ignorance and incomplete information where a partial preorder avoids comparing unrelated pieces of information. In such cases, an epistemic state can be represented by either a partial preorder on interpretations or a partially preordered belief base.

The revision of partially preordered information has been less investigated in the literature, however Lagrue and al. (Benferhat, Lagrue, and Papini 2005) pointed out that the KM's postulates are not appropriate for partial preorders and proposed a suitable definition of faithful assignment, called P-faithful assignment, a new set of postulates and a representation theorem. Some revision operations initially defined for total preorders, such as revision with memory and possibilistic revision have been sucessfully extended to partial preorders (Benferhat, Lagrue, and Papini 2002).

We propose a new framework for revising partially preordered information, in the sense of BRIE that extends the Removed Sets Revision to partially preordered information, called Partially Preordered Removed Sets Revision (PPRSR). Revising a partially preodered belief base according to the removed sets approach amounts to define, from a partial preorder on the belief base, a partial preorder on subsets of formulae to remove. In (Sérayet, Drap, and Papini 2009) we defined PPRSR operation based on the possibilistic preference on subsets of formulae (Benferhat, Lagrue, and Papini 2004), we now propose a new PPRSR operation stemming from the lexicographic preference on subsets of formulae (Yahi et al. 2008) and provide an efficient implementation thanks to Answer Set Programming. Combining lexicographic ordering and cardinality is a strategy used within the VENUS project where one of tasks is representing archeological survey information and managing this information in presence of inconsistencies. Data sets quality is not homogeneous, because the conditions are not the same on the whole archaeological site. When the data and the generic knowledge conflict, the strategy is to keep the maximal number of measures of highest quality. Moreover, a psychological study (Benferhat, Bonnefon, and Da Silva Neves 2004) has shown that the lexicographic inference based on lexicographic preference is more productive than the inclusion-preference and possibilistic inferences. The main contributions of this paper are the following:

- It proposes a revision operation for partially preordered belief bases stemming from the lexicographic preference on subsets of formulae (Yahi et al. 2008).
- It provides an implementation of this operation with ASP. The revision problem is translated into a logic program with answer set semantics and a one-to-one correspondence between removed sets and preferred answer sets is shown. The computation of answer sets is performed with any ASP solver.
- It shows that the revision with memory of partially preordered information can be captured within the PPRSR framework allowing for an efficient implementation with ASP.

The paper is organized as follows. After some notations, it reminds RSR, partial preorders, lexicographic preference, revision with memory and ASP. It presents the proposed lexicographic-based revision operation and shows how it captures the revision with memory. The encoding into logic programming with answer set semantics is detailled as well as the computation of answer sets thanks to ASP solvers. It then shows the one-to-one correspondence between removed sets and preferred answer sets. It finally illustrates how this revision operation could be applied in the context of the VENUS project before concluding.

Background and notations

Notations

In this paper we use propositional calculus, denoted by $\mathcal{L_{PC}}$, as knowledge representation language with usual connectives $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$. Let X be a set of propositional formulae, we denote by $Cons(X)$ the set of logical consequences of X, and $\bigvee_{1 \dots n}(X_i)$ denotes the disjunction of the sets of formulae X_i, $1 \leq i \leq n$. We denote by $\mathcal{W}$ the set of interpretations of $\mathcal{L_{PC}}$ and by $Mod(\psi)$ the set of models of a formula ψ, that is $Mod(\psi) = \{\omega \in \mathcal{W}, \omega \models \psi\}$ where $\models$ denotes the inference relation used for drawing conclusions. The symbol $\equiv$ denotes the logical equivalence.

Removed Sets Revision

We briefly recall the Removed Sets Revision approach. The Removed Sets Revision (Würbel, Jeansoulin, and Papini 2000) deals with the revision of a set of propositional formulae by a set of propositional formulae[1]. Let K and A be finite sets of clauses. The Removed Sets Revision focuses on the minimal subsets of clauses to remove from K, called *removed sets*, in order to restore the consistency of $K \cup A$. More formally: let K and A be two consistent sets of clauses such that $K \cup A$ is inconsistent. R a subset of clauses of K, is a removed set of $K \cup A$ iff (i) $(K \backslash R) \cup A$ is consistent; (ii) $\forall R' \subseteq K$, if $(K \backslash R') \cup A$ is consistent then $\mid R \mid \leq \mid R' \mid$[2]. Let denote by $\mathcal{R}(K \cup A)$ the collection of removed sets of $K \cup A$, the Removed Sets Revision (RSR) is defined as follows: let K and A be two consistent sets of clauses. The removed sets revision is defined by: $K \circ_{RSR} A =_{def} \bigvee_{R \in \mathcal{R}(K \cup A)} Cons((K \backslash R) \cup A)$. According to a semantic point of view, we consider $\mid \mathcal{NS}_K(\omega) \mid$ the number of clauses of K falsfied by an interpretation ω and we define a total preorder on interpretations: $\omega_i \leq_K \omega_j$ iff $\mid \mathcal{NS}_K(\omega_i) \mid \leq \mid \mathcal{NS}_K(\omega_j) \mid$. The Removed Sets Revision can be semantically defined by $Mod(K \circ_{RSR_{sem}} A) = min(Mod(A), \leq_K)$. It minimizes the number of clauses falsified by the models of A and $Mod(K \circ_{RSR} A) = Mod(K \circ_{RSR_{sem}} A)$. In case of prioritized belief bases, RSR has been extended to Prioritized Removed Sets Revision (PRSR) (Ben-Naim et al. 2004).

Partial preorders

A partial preorder, denoted by $\preceq$ on a set A is a reflexive and transitive binary relation. Let x and y be two members of A, the equality is defined by $x = y$ iff $x \preceq y$ and $y \preceq x$. The corresponding strict partial preorder, denoted by $\prec$, is such that, $x \prec y$ iff $x \preceq y$ holds but $x = y$ does not hold. We denote by $\sim$ the incomparability relation $x \sim y$ iff $x \preceq y$ does not hold nor $y \preceq x$. The set of minimal elements of A with respect to $\prec$, denoted by $Min(A, \prec)$, is defined as: $Min(A, \prec) = \{x \in A, \nexists y \in A : y \prec x\}$.

Generally, epistemic states are represented by total preorders on interpretations, however, as mentioned in the introduction, in case of partial ignorance, the agent is unable to compare all situations between them and a partial preorder seems to be more suitable to represent epistemic states.

Let Ψ be an epistemic state and $Bel(\Psi)$ its corresponding belief set, Ψ is first represented by a partial preorder on interpretations, denoted by $\preceq_\Psi$. In (Benferhat, Lagrue, and Papini 2005), a suitable definition of faithful assignment is given: let $Mod(Bel(\Psi)) = min(\mathcal{W}, \preceq_\Psi)$, $\preceq_\Psi$ is a P-faifhful assignment if (1) if $\omega, \omega' \models Bel(\Psi)$ then

[1] We consider propositional formulae in their equivalent conjunctive normal form (CNF).

[2] $\mid R \mid$ denotes the number of clauses of R.

$\omega \prec_\Psi \omega'$ does not hold, (2) if $\omega' \not\models Bel(\Psi)$, then there exists ω such that $\omega \models Bel(\Psi)$ and $\omega \prec_\Psi \omega'$, (3) if $\Psi = \Phi$ then $\preceq_\Psi = \preceq_\Phi$. Moreover, (Benferhat, Lagrue, and Papini 2005) gives a set of postulates an operation $\circ$ has to satisfy and a representation theorem such that $Mod(Bel(\Psi \circ \mu)) = min(Mod(\mu), \preceq_\Psi)$. An alternative syntactic but equivalent representation of an epistemic state, Ψ is a partially preordered belief base, denoted by $(\Sigma, \preceq_\Sigma)$, where Σ is a set of propositional formulae, and $\preceq_\Sigma$ is a partial preorder on the formulae of Σ.

Lexicographic preference

Several ways for defining a preference relation on subsets of formulae of Σ, from a partial preorder $\preceq_\Sigma$ have been proposed: inclusion-based (Junker and Brewka 1989), possibilistic (Benferhat, Lagrue, and Papini 2004), lexicographic (Yahi et al. 2008) preferences. In this paper we focus on the lexicographic preference which extends the lexicographic preorder initially defined for totally preordered belief bases to partially preordered belief bases. The belief base Σ is partitioned such that $\Sigma = E_1 \cup \ldots \cup E_n$ $(n \geq 1)$ where each subset E_i represents an equivalence class of Σ with respect to $=_\Sigma$ which is an equivalence relation. A preference relation between the equivalence classes E_i's, denoted by $\prec_s$ is defined by $E_i \prec_s E_j$ iff $\exists \varphi \in E_i$, $\exists \varphi' \in E_j$ such that $\varphi \prec_\Sigma \varphi'$. This partition can be viewed as a generalization of the idea of stratification defined for totally preordered belief bases. A lexicographic preference relation between the consistent subbases of a partially preordered belief base $(\Sigma, \preceq_\Sigma)$, denoted by $\preceq_\Delta$, is defined as follows:

Definition 1 *(Yahi et al. 2008) Let $(\Sigma, \preceq)$ be a partially preordered belief base and let A and B be two consistent subbases of Σ. Then, A is said to be lexicographically preferred to B, denoted by $A \preceq_\Delta B$, iff $\forall i, 1 \leq i \leq n$:*
if $|E_i \cap B| > |E_i \cap A|$ then $\exists j, 1 \leq j \leq n$ such that
$|E_j \cap A| > |E_j \cap B|$ and $E_j \prec_s E_i$.

In our approach, according to the Removed Sets strategy, we adopt a dual point of view in the sense that we want to prefer the subsets of formulae to remove. We rephrase the lexicographic preference defined in (Yahi et al. 2008): let $\preceq_\Sigma$ be a partial preorder on Σ, $Y \subseteq \Sigma$ and $X \subseteq \Sigma$. Y is said to be lexicographically preferred to X, denoted by $Y \trianglelefteq_\Delta X$, iff $X \preceq_\Delta Y$.

We now briefly remind the extension of the semantic revision with memory to partial preorders (Benferhat, Lagrue, and Papini 2002). Let Ψ be an epistemic state, represented by a partial preorder on interpretations denoted by $\preceq_\Psi$, The revision with memory of Ψ by a propositional formula μ leads to the epistemic state $\Psi \circ_\triangleright \mu$, represented by the partial preorder $\preceq_{\Psi \circ_\triangleright \mu}$ which preserves the relative ordering between the models of μ as well as the relative ordering between the countermodels of μ. More formally $\Psi \circ_\triangleright \mu$ corresponds to the following partial preorder: (i) if $\omega, \omega' \in Mod(\mu)$ then $\omega \preceq_{\Psi \circ_\triangleright \mu} \omega'$ iff $\omega \preceq_\Psi \omega'$, (ii) if $\omega, \omega' \notin Mod(\mu)$ then $\omega \preceq_{\Psi \circ_\triangleright \mu} \omega'$ iff $\omega \preceq_\Psi \omega'$, (iii) if $\omega \in Mod(\mu)$ and $\omega' \notin Mod(\mu)$ then $\omega \prec_{\Psi \circ_\triangleright \mu} \omega'$.

Answer sets

A *normal logic program* is a set of rules of the form $c \leftarrow a_1, \ldots, a_n, not\ b_1, \ldots, not\ b_m$ where $c, a_i (1 \leq i \leq n), b_j (1 \leq j \leq m)$ are propositional atoms and the symbol not stands for *negation as failure*. A rule is a fact if $n = m = 0$, it is a basic rule if $m = 0$. For a rule r like above, we introduce $head(r) = c$ and $body(r) = \{a_1, \cdots, a_n, b_1, \cdots, b_m\}$. Furthermore, let $body^+(r) = \{a_1, \cdots, a_n\}$ denotes the set of positive body atoms and $body^-(r) = \{b_1, \cdots, b_m\}$ the set of negative body atoms, and $body(r) = body^+(r) \cup body^-(r)$. By extension, a basic program is a program containing only basic rules. Let r be a rule, r^+ denotes the rule $head(r) \leftarrow body^+(r)$, obtained from r by deleting all negative body atoms in the body of r.

A set of atoms X is *closed under* a basic program P iff for any rule $r \in P$, $head(r) \in X$ whenever $body(r) \subseteq X$. The smallest set of atoms which is closed under a basic program P is denoted by $CN(P)$. The *reduct* or Gelfond-Lifschitz transformation (Gelfond and Lifschitz 1988), P^X of a program P relatively to a set X of atoms is defined by $P^X = \{r^+ \mid r \in P\ and\ body^-(r) \cap X = \emptyset\}$. A set of atoms X is an *answer set* of P iff $CN(P^X) = X$.

Lexicographic-based PPRSR

Let Ψ be an epistemic state, syntactically represented by a partially preordered belief bases denoted by $(\Sigma, \preceq_\Sigma)$ where Σ is a consistent set of propositinal formulae and $\preceq_\Sigma$ is a partial preorder on Σ.

Syntactic approach The lexicographic-based revision of Ψ by a propositional formula μ is defined as follows:

Definition 2 *Let Ψ be an epistemic state represented by $(\Sigma, \preceq_\Sigma)$. the revision of Ψ by a formula μ leads to the revised epistemic state denoted by $\Psi \circ_{\trianglelefteq_\Delta} \mu$ represented by a partially preordered belief base $(\Sigma \circ_{\trianglelefteq_\Delta} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_\Delta} \mu})$ where*

- $\Sigma \circ_{\trianglelefteq_\Delta} \mu = \Sigma \cup \{\mu\}$
- $\preceq_{\Sigma \circ_{\trianglelefteq_\Delta} \mu}$: *$(i)$ $\forall \psi \in \Sigma$, $\mu \prec_{\Sigma \circ_{\trianglelefteq_\Delta} \mu} \psi$ and*
  (ii) $\forall \psi, \phi \in \Sigma$, $\psi \preceq_{\Sigma \circ_{\trianglelefteq_\Delta} \mu} \phi$ iff $\psi \preceq_\Sigma \phi$

Since $\Sigma \cup \{\mu\}$ may be inconsistent, we have to provide the consistent belief set, denoted by $Bel(\Psi \circ_{\trianglelefteq_\Delta} \mu)$, corresponding to the revised epistemic state. In order to syntactically compute $Bel(\Psi \circ_{\trianglelefteq_\Delta} \mu)$ we focus on the preferred subsets of formulae to remove from Σ to restore consistency. We first define the potential removed sets as follows:

Definition 3 *Let $(\Sigma, \preceq_\Sigma)$. Let μ be a formula s. t. $\Sigma \cup \{\mu\}$ is inconsistent. R, a subset of formulae of Σ, is a potential removed set of $\Sigma \cup \{\mu\}$ iff $(\Sigma \backslash R) \cup \{\mu\}$ is consistent.*

We introduce a running example that we use from now on.

Example 1 *Let $\Sigma = \{a, b, \neg c\}$ and $\preceq_\Sigma$ such that:*
$$\begin{array}{ccc} & b & \\ \swarrow & & \searrow \\ a & & \neg c \end{array}$$
where $a \leftarrow b$ means that $a \prec_\Sigma b$. We revise Σ by $\mu = (\neg a \vee b) \wedge (\neg a \vee c)$. $\Sigma \cup \{\mu\}$ is inconsistent. The potential removed sets are $R_0 = \{a\}$, $R_1 = \{a, b\}$, $R_2 = \{\neg c\}$, $R_3 = \{a, \neg c\}$, $R_4 = \{b, \neg c\}$ and $R_5 = \{a, b, \neg c\}$.

Among them, we want to prefer the potential removed sets which allow us to remove the formulae that are not preferred according to $\preceq_\Sigma$. This leads to define a partial preorder on subsets of formulae of Σ using the lexicographic comparator $\trianglelefteq_\triangle$. We now generalize the notion of Removed Sets to subsets of partially preordered formulae. We denote by $\mathcal{R}_\triangle(\Sigma \cup \{\mu\})$ the set of removed sets of $\Sigma \cup \{\mu\}$.

Definition 4 *Let $(\Sigma, \preceq_\Sigma)$ and let μ be a formula s. t. $\Sigma \cup \{\mu\}$ is inconsistent. $R \subseteq \Sigma$ is a removed set of $\Sigma \cup \{\mu\}$ iff*

1. *R is a potential removed set.*
2. *$\nexists R' \subseteq \Sigma$ s. t. $(\Sigma \backslash R') \cup \{\mu\}$ is consistent and $R' \triangleleft_\triangle R$.*

Example 2 *The partial preorder on potential removed sets from example 1 is:*

$$\begin{tikzcd}[column sep=small]
& R_5 \arrow[d] & \\
& R_3 \arrow[dl] \arrow[dr] & \\
R_1 \arrow[d] & & R_4 \arrow[d] \\
R_0 & & R_2
\end{tikzcd}$$

$R_3 \leftarrow R_5$ means that $R_3 \triangleleft_\triangle R_5$. We have $\nexists R' \subseteq \Sigma$ such that $(\Sigma \backslash R') \cup \{\mu\}$ is consistent and $R' \triangleleft_\triangle R_0$ and $R' \triangleleft_\triangle R_2$.

Using the lexicographic preference, the belief set $Bel(\Psi \circ_{\trianglelefteq_\triangle} \mu)$ corresponding to the revised epistemic state is defined as follows:

Definition 5 *Let $(\Sigma, \preceq_\Sigma)$ be the syntactic representation of Ψ and let μ be a formula, the belief set corresponding to the revised epistemic state $\Psi \circ_{\trianglelefteq_\triangle} \mu$ is $Bel(\Psi \circ_{\trianglelefteq_\triangle} \mu) = \bigvee_{R \in R_\triangle(\Sigma \cup \{\mu\})} Cons((\Sigma \backslash R) \cup \{\mu\})$.*

Example 3 *According to the example 1, the revision of $(\Sigma, \preceq_\Sigma)$ by μ using the lexicographic preference gives:*

- $\Sigma \circ_{\trianglelefteq_\triangle} \mu = \{a, b, \neg c, (\neg a \vee b) \wedge (\neg a \vee c)\}$
- $Bel(\Psi \circ_{\trianglelefteq_\triangle} \mu) = Cons(\{b, \neg c, (\neg a \vee b) \wedge (\neg a \vee c)\}) \bigvee Cons(\{a, b, (\neg a \vee b) \wedge (\neg a \vee c)\})$
- $\preceq_{\Sigma \circ_{\trianglelefteq_\triangle} \mu}$:

$$\begin{tikzcd}[column sep=small]
& b \arrow[dl] \arrow[dr] & \\
a \arrow[dr] & & \neg c \arrow[dl] \\
& (\neg a \vee b) \wedge (\neg a \vee c) &
\end{tikzcd}$$

Semantic approach According to a semantic point of view, the epistemic state Ψ can be equivalently represented by a partial preorder on interpretations such that $Mod(Bel(\Psi))$ is minimal in this preorder. Let ω be an interpretation, $F_\Sigma(\omega)$ denotes the set of formulae of Σ falsified by ω. We now construct a partial preorder on interpretations from $(\Sigma, \preceq_\Sigma)$ as follows :

Definition 6 $\forall \omega, \omega' \in \mathcal{W}$, *$\omega \preceq^\triangle_\Psi \omega'$ iff $F_\Sigma(\omega) \trianglelefteq_\triangle F_\Sigma(\omega')$.*

Using this definition, the semantic representation of Ψ is such that $Mod(Bel(\Psi)) = min(\mathcal{W}, \preceq^\triangle_\Psi)$. Moreover the following proposition holds.

Proposition 1 *Let Ψ be an epistemic state and $\preceq^\triangle_\Psi$ be a partial preorder on $\mathcal{W}$ associated to Ψ. Then, $\preceq^\triangle_\Psi$ is a P-faithful assignment.*

The semantic counterpart of our lexicographic-based revision operation, denoted by $\circ_{\trianglelefteq^{sem}_\triangle}$, is defined as follows:

Definition 7 *Let Ψ be an epistemic state and let μ be a formula. $Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu)) = min(Mod(\mu), \preceq^\triangle_\Psi)$.*

The semantic representation of the revised epistemic state is $\preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu}$ with $\preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu}$ defined by $\omega \preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu} \omega'$ iff $F_{\Sigma \circ_{\trianglelefteq_\triangle} \mu}(\omega) \trianglelefteq_\triangle F_{\Sigma \circ_{\trianglelefteq_\triangle} \mu}(\omega')$. The following proposition gives the equivalence between the semantic and the syntactic lexicographic-based revision operation.

Proposition 2 *Let $(\Sigma, \preceq_\Sigma)$ and let μ be a formula. $Mod(Bel(\Psi \circ_{\trianglelefteq_\triangle} \mu)) = Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu))$.*

According to Proposition 1, the revision operation $\circ_{\trianglelefteq_\triangle}$ satisfies the postulates $P_1 - P_7$ proposed in (Benferhat, Lagrue, and Papini 2005) that extend the KM-postulates to the revision of partially preordered belief bases.

Example 4 *Let $(\Sigma, \preceq_\Sigma)$ from example 1. We construct a partial preorder on $\mathcal{W}$ from definition 6 and the lexicographic preference. The sets of formulae of Σ falsified by the interpretations is illustrated in Table 1 and the partial preorder $\preceq^\triangle_\Psi$ is given by the Figure 1 (a). Therefore $\preceq^\triangle_\Psi$ is the semantic representation of Ψ and is such that $Mod(Bel(\Psi)) = min(\mathcal{W}, \preceq^\triangle_\Psi) = \{\omega_6\}$. Let $(\Sigma \circ_{\trianglelefteq_\triangle} \mu, \preceq_{\Sigma \circ_{\trianglelefteq_\triangle} \mu})$ be the syntactic representation of the epistemic state Ψ revised by μ. As previously, we construct a new partial preorder on the interpretations. The sets of formulae of $\Sigma \circ_{\trianglelefteq_\triangle} \mu$ falsified by the interpretations are illustrated in Table 1 and the partial preorder $\preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu}$ is given by the Figure 1 (b). Therefore $\preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu}$ is the semantic representation of Ψ revised by μ and with proposition 2 is such that $Mod(Bel(\Psi \circ_{\trianglelefteq_\triangle} \mu)) = min(Mod(\mu), \preceq^\triangle_\Psi) = \{\omega_2, \omega_7\}$.*

ω_i	a	b	c	$F_\Sigma(\omega_i)$	$F_{\Sigma \circ_{\trianglelefteq_\triangle} \mu}(\omega_i)$
ω_0	$\neg a$	$\neg b$	$\neg c$	$\{a, b\}$	$\{a, b\}$
ω_1	$\neg a$	$\neg b$	c	$\{a, b, \neg c\}$	$\{a, b, \neg c\}$
ω_2	$\neg a$	b	$\neg c$	$\{a\}$	$\{a\}$
ω_3	$\neg a$	b	c	$\{a, \neg c\}$	$\{a, \neg c\}$
ω_4	a	$\neg b$	$\neg c$	$\{b\}$	$\{b, (\neg a \vee b) \wedge (\neg a \vee c)\}$
ω_5	a	$\neg b$	c	$\{b, \neg c\}$	$\{b, \neg c, (\neg a \vee b) \wedge (\neg a \vee c)\}$
ω_6	a	b	$\neg c$	$\emptyset$	$\{(\neg a \vee b) \wedge (\neg a \vee c)\}$
ω_7	a	b	c	$\{\neg c\}$	$\{\neg c\}$

Table 1: The sets of formulae falsified by the interpretations

$$\begin{tikzcd}[column sep=small]
& \omega_1 \arrow[d] & \\
& \omega_3 \arrow[dl] \arrow[dr] & \\
\omega_0 \arrow[d] & & \omega_5 \arrow[d] \\
\omega_2 \arrow[dr] & & \omega_7 \arrow[dl] \\
& \omega_4 \arrow[d] & \\
& \omega_6 &
\end{tikzcd}$$

(a) $\preceq^\triangle_\Psi$

$$\begin{tikzcd}[column sep=small]
& \omega_5 \arrow[d] & \\
& \omega_4 \arrow[d] & \\
& \omega_6 \arrow[d] & \\
& \omega_1 \arrow[d] & \\
& \omega_3 \arrow[dl] \arrow[dr] & \\
\omega_0 \arrow[d] & & \omega_7 \\
\omega_2 & &
\end{tikzcd}$$

(b) $\preceq^\triangle_{\Psi \circ_{\trianglelefteq^{sem}_\triangle} \mu}$

Figure 1: Partial preorders between interpretations.

The PPRSR framework can capture the revision with memory and the following proposition holds.

Proposition 3 *Let $\circ_{\triangleright}$ be the revision with memory operation. $\forall \omega, \omega' \in \mathcal{W}$, $\omega \preceq^{\triangle}_{\Psi \circ_{\trianglelefteq^{sem}_{\triangle}} \mu} \omega'$ iff $\omega \preceq_{\Psi \circ_{\triangleright} \mu} \omega'$.*

The resulting partial preorder on interpretations using the $\circ_{\trianglelefteq^{sem}_{\triangle}}$ revision operation is the same than the one holding after using the memory revision operation. Therefore, we have $Mod(Bel(\Psi \circ_{\trianglelefteq^{sem}_{\triangle}} \mu)) = Mod(Bel(\Psi \circ_{\triangleright} \mu))$. Thanks to the proposition 3, the proposed lexicographic-based revision operation satisfies the Darwiche and Pearl's postulate (Darwiche and Pearl 1997) for iterated revision.

Example 5 *Consider the example above $Mod(\mu) = \{\omega_0, \omega_1, \omega_2, \omega_3, \omega_7\}$ Figures 1 (a) and 1 (b) illustrate that the relative ordering between the models of μ is preserved as well as the relative ordering between the countermodels of μ. In particular, the incomparabilities between ω_0 and ω_7 on one hand, and between ω_2 and ω_7 on the other hand are preserved.*

Encoding $\circ_{\trianglelefteq_{\triangle}}$ in Answer Set Programming

In order to compute the removed sets, we propose an extension of the methods proposed by (Hué, Würbel, and Papini 2008) and (Ben-Naim et al. 2004) for revising partially preordered belief bases. After translating our revision problem into a logic program with answer sets semantics, denoted by $\Pi_{\Sigma \cup \{\mu\}}$, we then define a partial preorder between answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and we show a correspondence between removed sets of $\Sigma \cup \{\mu\}$ and preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$.

Let Σ be a set of partially preordered formulae and μ be a formula such that $\Sigma \cup \{\mu\}$ is inconsistent. The set of all positive, resp. negative literals of $\Pi_{\Sigma \cup \{\mu\}}$ is denoted by V^+, resp. V^-. The set of rule atoms representing formulae is defined by $R^+ = \{r_f | f \in \Sigma\}$ and $F_O(r_f)$ represents the formula of Σ corresponding to r_f in $\Pi_{\Sigma \cup \{\mu\}}$, namely $\forall r_f \in R^+, F_O(r_f) = f$. This translation requires the introduction of intermediary atoms representing subformulae. We denote by ρ^j_f the intermediary atom representing f^j which is a subformula of $f \in \Sigma$.

1. In step 1, we introduce rules in order to build a one-to-one correspondence between answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and interpretations of V^+. For each atom, $a \in V^+$ two rules are introduced: $a \leftarrow not\ a'$ and $a' \leftarrow not\ a$ where $a' \in V^-$ is the negative atom corresponding to a.
2. In step 2, we introduce rules in order to exclude the answer sets S corresponding to interpretations which are not models of $(\Sigma \backslash F) \cup \{\mu\}$ with $F = \{f | r_f \in S\}$. According to the syntax of f, the following rules are introduced:
   - If $f =_{def} a$ then $r_f \leftarrow not\ a$ is introduced;
   - If $f =_{def} \neg f^1$ then $r_f \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $f =_{def} f^1 \vee \ldots \vee f^m$ then $r_f \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $f =_{def} f^1 \wedge \ldots \wedge f^m$ then it is necessary to introduce several rules: $\forall 1 \leq j \leq m, r_f \leftarrow \rho_{f^j}$.
3. Step 3 rules out answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ which correspond to interpretations which are not models of μ. According to the syntax of μ, the following rules are introduced:
   - If $\mu =_{def} a$ then $false \leftarrow not\ a$ is introduced;
   - If $\mu =_{def} \neg f^1$ then $false \leftarrow not\ \rho_{f^1}$ is introduced;
   - If $\mu =_{def} f^1 \vee \ldots \vee f^m$ then $false \leftarrow \rho_{f^1}, \ldots, \rho_{f^m}$ is introduced;
   - If $\mu =_{def} f^1 \wedge \ldots \wedge f^m$ then $\forall 1 \leq j \leq m$, $false \leftarrow \rho_{fj}$ are introduced.

   $contradiction \leftarrow false, not\ contradiction$ is introduced in order to rule out $false$ from the models of μ.

Example 6 *For the previous example, the logic program $\Pi_{\Sigma \cup \{\mu\}}$ is the following:*

$a \leftarrow not\ a'$	$c \leftarrow not\ c'$	$r_{\neg c} \leftarrow c$	$\rho_{\neg a \vee c} \leftarrow \rho_{\neg a}, \rho_c$
$a' \leftarrow not\ a$	$c' \leftarrow not\ c$	$false \leftarrow \rho_{\neg a \vee b}$	$\rho_{\neg a} \leftarrow a$
$b \leftarrow not\ b'$	$r_b \leftarrow not\ b$	$false \leftarrow \rho_{\neg a \vee c}$	$\rho_{\neg b} \leftarrow not\ b$
$b' \leftarrow not\ b$	$r_a \leftarrow not\ a$	$\rho_{\neg a \vee b} \leftarrow \rho_{\neg a}, \rho_b$	$\rho_{\neg c} \leftarrow not\ c$

$contradiction \leftarrow false, not\ contradiction$

$S(\Pi_{\Sigma \cup \{\mu\}})$ denotes the set of answer sets of $\Pi_{\Sigma \cup \{\mu\}}$. Each answer set S corresponds to an interpretation of $\Sigma \cup \{\mu\}$, $I_S = \{a | a \in S\} \cup \{\neg a | a' \in S\}$ and each interpretation of $\Sigma \cup \{\mu\}$ corresponds to several potential removed sets. Therefore, the following result holds.

Proposition 4 *Let ρ a rule atom or an intermediary atom. $\rho \in CN(\Pi^S_{\Sigma \circ \mu})$ iff $I_S \not\models F_O(R^+ \cap S)$.*

The correspondence between answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and interpretations of $(\Sigma \backslash F_O(R^+ \cap S)) \cup \{\mu\}$ is given in the following proposition:

Proposition 5 *Let $(\Sigma, \preceq_\Sigma)$. Let $S \subseteq V$ be a set of atoms. S is an answer set of $\Pi_{\Sigma \cup \{\mu\}}$ iff S corresponds to an interpretation I_S of V^+ which satisfies $(\Sigma \backslash F_O(R^+ \cap S)) \cup \{\mu\}$.*

Example 7 *The answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ are : $S_0 = \{b, a, c, r_{\neg c}\}$, $S_1 = \{b, c, a', r_a, r_{\neg c}\}$, $S_2 = \{c, a', b', r_a, r_b, r_{\neg c}\}$, $S_3 = \{b, c', a', r_a\}$ and $S_4 = \{c', a', b', r_a, r_b\}$.*

We introduce new preference relations between answer sets according to a partial preorder. We define a preferred answer set according to the comparator $\trianglelefteq_\triangle$. We denote by $S_\triangle(\Pi_{\Sigma \cup \{\mu\}})$ the set of preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$, more formally:

Definition 8 *Let $(\Sigma, \preceq_\Sigma)$. Let μ be a formula s. t. $\Sigma \cup \{\mu\}$ is inconsistent, $S \in S(\Pi_{\Sigma \cup \{\mu\}})$. S is a preferred answer set of $\Pi_{\Sigma \cup \{\mu\}}$ iff $\nexists S' \in S(\Pi_{\Sigma \cup \{\mu\}})$, $F_O(S' \cap R^+) \triangleleft_\triangle F_O(S \cap R^+)$.*

The one-to-one correspondence between preferred answer sets of $\Pi_{\Sigma \cup \{\mu\}}$ and the removed sets is given by the following proposition:

Proposition 6 *Let $(\Sigma, \preceq_\Sigma)$. Let μ be a formula s. t. $\Sigma \cup \{\mu\}$ is inconsistent. X is a removed set of Σ iff there exists a preferred answer set S of $\Pi_{\Sigma \cup \{\mu\}}$ s. t. $F_O(R^+ \cap S) = X$.*

Example 8 *The partial preorder on the answer sets is:*

$$\begin{array}{ccc} & S_2 & \\ & \downarrow & \\ & S_1 & \\ \swarrow & & \searrow \\ S_0 & & S_4 \\ & & \downarrow \\ & & S_3 \end{array}$$

where $S_1 \leftarrow S_2$ means that: $F_O(S_1 \cap R^+) \triangleleft_\triangle F_O(S_2 \cap R^+)$. Moreover, we have $F_O(S_0 \cap R^+) = \{\neg c\}$ and $F_O(S_2 \cap R^+) = \{a\}$ which correspond to removed sets R_0 and R_2 found in the previous section.

Regarding the implementation, CLASP (Gebser et al. 2007) gives us the answer sets of $\Pi_{\Sigma \cup \{\mu\}}$. From them, our method requires to construct a partially preorder between them using the lexicographic comparator $\trianglelefteq_{\Delta}$ to obtain the preferred answer sets corresponding to removed sets. This step is not yet implemented in ASP. We used a java program to partially preorder the answer sets to obtain the preferred answer sets. Given that the lexicographic comparator satisfies the monotony property, ($\forall X, Y \subseteq \Sigma$, if $Y \subseteq X$ then $Y \trianglelefteq_{\Delta} X$), it is sufficient to compare the answer sets which are minimal according to the inclusion. This is why we compute the removed sets rather than the maximal consistent subbases. Moreover, the determination of the minimal answer sets according to this partial preorder does not increase the cost since the complexity of CLASP is similar to the complexity of the SAT problem.

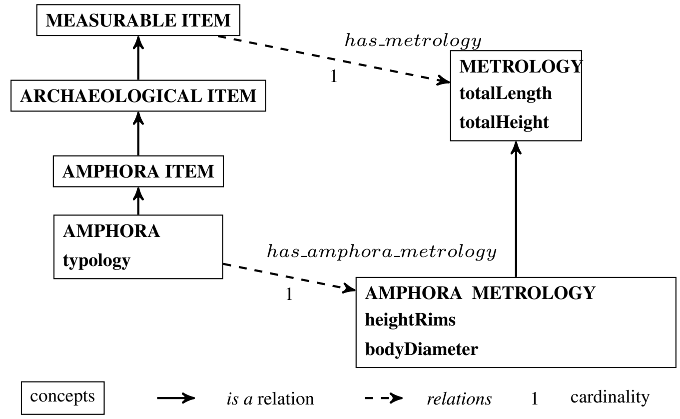

Figure 2: Extract of the application ontology

VENUS application

We now present how our lexicographic-based revision operation could be applied in a real context. The european VENUS project (Virtual ExploratioN of Underwater Sites) no (IST-034924)[3] aims at providing scientific methodologies and technological tools for the virtual exploration of deep underwater archaeology sites. In this context, technologies like photogrammetry are used for data acquisition and the knowledge about the studied objects is provided by both archaeology and photogrammetry. We constructed an application ontology in (Sérayet et al. 2009) from a domain which describes the amphorae (the studied artefacts) and from a task, the data acquisition process by photogrammetry. This ontology consists of a set of concepts, relations, attributes and constraints like domain constraints: an amphora must have only one typology and for example, this typology is either short Dressel 2-4 or long Dressel 2-4. Our knowledge base contains our ontology and data sets coming from the photogrammetric process. The ontology represents the generic knowlegde which is preferred to the data sets. The data sets quality is not homogeneous, because the conditions are not the same on the whole archaeological site. The quality of the data sets directly comes from the quality of the pictures since within the photogrammetric process the 2D measures are performed from pictures. It is relevant to use several data sets for the same amphora which is measured several times. A preference relation is defined on the data sets according to their quality. We only consider a small part of the ontology (Figure 2) and some data sets for the lack of space where the knowledge base is expressed in propositional logic.

We use the following propositional variables: m_i for the measurable item, ar_i for the archaeological item, a_i for the amphora item, a for the amphora, m_1, m_2 for the metrologies, d_s for the short Dressel $2-4$ typology, d_l for the long Dressel $2-4$ typology, h_{m_1}, h_{m_2} for has_metrology, h_1, h_2 for the total heights, l_1, l_2 for the total lengths. The propositional translation of the extract of the ontology can be resumed by the set of formulae: $G = \{a \rightarrow a_i \wedge (d_c \vee d_l),\ a_i \rightarrow ar_i,\ ar_i \rightarrow m_i,\ m_i \rightarrow h_{m_1} \vee h_{m_2},\ h_{m_1} \rightarrow m_1,\ h_{m_2} \rightarrow m_2,\ m_1 \rightarrow l_1 \wedge h_1,\ m_2 \rightarrow l_2 \wedge h_2, (d_s \vee d_l) \wedge (\neg d_s \vee \neg d_l)\}$. Then we have two data sets. The first data set $\{a, d_c, l_1, h_1\}$ lead to the instance denoted by $I_1 = \{a,\ a_i,\ ar_i,\ d_c,\ m_1,\ m_i,\ h_{m_1},\ l_1,\ h_1\}$ and the second one is $\{a, d_l, l_2, h_2\}$ lead to the instance denoted by $I_2 = \{a,\ a_i,\ ar_i,\ d_l,\ m_2,\ m_i,\ h_{m_2},\ l_2,\ h_2\}$. By hypothesis, the ontology and the constraints which are also called the generic knowledge cannot be modified. Moreover, we consider that the second data set has higher quality than the first one. We revised the first data set $\Sigma = I_1 \backslash (I_1 \cap I_2)$ by $M = G \cup I_2$ where G is the generic knowledge and I_2 is the second data set and the revised preorder is represented by Figure 3. We obtain: $Bel(\Psi \circ_{\trianglelefteq_C} M) = Cons((\Sigma \backslash R) \cup M)$ with $R = \{d_s\}$. The revision presented in a previous section is the first step of the revision to apply in the VENUS context. Indeed, the revision could be defined as follows:

- $Bel(\Psi \circ_{\trianglelefteq_C} M) = \bigvee_{R \in \mathcal{R}_C(\Sigma \cup M)} Cons((\Sigma \backslash R) \cup M)$ with $\Sigma = I_1 \backslash (I_1 \cap I_2)$ and $M = G \cup I_2$.
- $\preceq_{\Sigma \circ_{\trianglelefteq_C} M}$: (i) $\forall \psi, \phi \in M$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} M} \phi$ iff $\psi \preceq_M \phi$; (ii) $\forall \psi, \phi \in \Sigma$, $\psi \prec_{\Sigma \circ_{\trianglelefteq_C} M} \phi$ iff $\psi \preceq_{\Sigma} \phi$; (iii) $\forall \psi \in \Sigma$, $\mu \in M$, $\mu \prec_{\Sigma \circ_{\trianglelefteq_C} M} \psi$.

Conclusion

This paper presents a lexicographic-based revision operation within the framework of Partially Preordered Removed Sets Revision for revising partially preordered information. It shows that the extension of revision with memory to partial preorders can be captured within this framework. It also shows that this revision operation can be successfully encoded into ASP and proposes an implementation stemming for any ASP solvers. It illustrates how this revision operation could be applied within the context of the VENUS european project dealing with archaeological information. In a future work, we have to conduct an experimental study. We also have to deeper investigate the use of ASP solver statements in order to directly define a partial preorder between answer sets.

[3] http://www.venus-project.eu

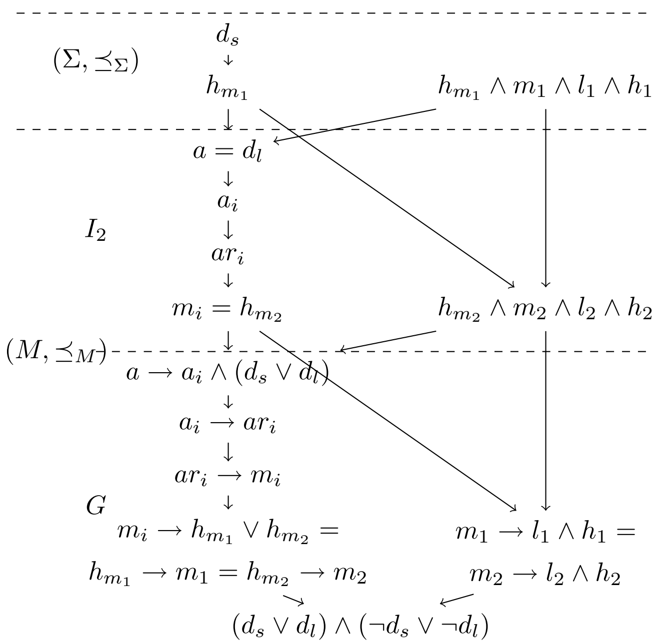

Figure 3: $\preceq_{\Sigma \circ_{\unlhd_c} M}$

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